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Symmetry breaking induced by top quark loops from a model without scalar mass.

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Abstract

Considering the standard model as an effective electroweak theory, in which we have no scalar mass term in the Higgs potential ($\mu^2 = 0$), we show that the spontaneous symmetry breaking of $SU(2)_L \times U(1)$ can be induced by top loops. The Higgs boson mass obtained is smaller than 300 GeV.

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In this letter, to implement the spontaneous symmetry breaking of $SU(2)_L \times U(1)$, we consider an effective model, i.e. a model with a cut-off at some physical scale Λ . The lagrangian of this model is the lagrangian of the standard model with the following Higgs potential

$$V(\Phi) = \lambda(\Phi^\dagger \Phi)^2 \quad (1)$$

where Φ is the scalar doublet

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_3 \\ \phi_1 + i\phi_4 \end{pmatrix} \quad (2)$$

Consequently we assume a scalar quartic interaction ($\lambda \neq 0$) but no scalar mass term ($\mu^2 = 0$). The Higgs potential of Eq.(1), with $\mu^2 = 0$, cannot produce the spontaneous symmetry breaking (SSB) at tree level. Nevertheless we show that in this model the one loop effects coming from the top Yukawa force induce the SSB of $SU(2)_L \times U(1)$. In this way the SSB induced by top loops is responsible (and this is the important point) for all the particles masses, i.e. the Higgs boson mass (which is expected to be smaller than 300 GeV) as well as the fermions and gauge bosons masses.

Before considering the model in detail, it is useful to situate it in its context. The mechanism of SSB by loop corrections considered here, which is based on the effective potential formalism, has been introduced by S. Coleman and E. Weinberg (CW) in ref.[1]. However the version of the model used here to implement this mechanism is different from the one considered by CW in ref.[1]. CW have considered a renormalizable model where the scalar renormalized mass μ_R is put to be 0 in the renormalized Higgs potential (with μ_R^2 defined as $(\partial^2 V_{eff}(\phi)/\partial\phi^2)|_{\phi=0}$ with $V_{eff}(\phi)$ the effective potential and ϕ the neutral scalar field). CW have shown that the one loop corrections coming from the gauge sector (the top Yukawa coupling was neglected in ref.[1]) induce a SSB. After adjusting λ so that the effective potential has an extremum at $\phi = v \simeq 246$ GeV the corresponding square Higgs mass m_H^2 (defined by the expression $(\partial^2 V_{eff}(\phi)/\partial\phi^2)|_{\phi=v}$) has been obtained to be $m_H^2 = (3m_W^4 + \frac{3}{2}m_Z^4)/4\pi^2 v^2 \simeq (9.8 \text{ GeV})^2$. As is well-known this value is excluded by the present experimental lower limit $m_H \gtrsim 65$ GeV (see for example ref. [2]). In addition, when

we take in the CW model the top Yukawa coupling contribution, the corresponding value obtained is $m_H^2 = (3m_W^4 + \frac{3}{2}m_Z^4 - 2N_c m_t^4)/4\pi^2 v^2$ (with m_t the top quark mass and N_c the number of colors). With the present experimental value $m_t = (180 \pm 12)$ GeV (see for example ref. [2] from the experimental results of ref.[3]) we obtain $m_H^2 = -(50 \text{ GeV})^2$ which means that the extremum in $\phi = v$ is not a minimum but a maximum and therefore the CW model does not work at all.

In the effective model considered in the present letter the top Yukawa contribution is dominant and large as in the CW renormalizable model (because m_t is large) but contrary to the CW model case is responsible for a minimum in $\phi = v$ i.e. for SSB. Therefore to consider the standard model as an effective theory (with no mass term in eq.(1)) instead of a renormalizable theory (with $\mu_R^2 = 0$) implies a different situation for SSB. It is interesting to note that in the effective theory the meaning of “no mass” for the scalar fields before SSB is not the same as in the renormalizable theory. In the renormalizable theory of CW there is a quadratic counterterm in the scalar fields, which means a mass term in the bare lagrangian. The mass which is put to be zero in the CW renormalizable model is the renormalized mass which is renormalization scheme dependant. The choice of the scheme where the renormalized mass is put to be zero must consequently be justified by a physical argument. In the effective theory “no mass” before SSB simply means no quadratic term in the scalar part of the effective lagrangian, i.e. no quadratic term coming from the physics at the scale Λ .

The model considered in this letter has also to be put in relation with the two models (one renormalizable and one effective) considered in ref.[4]. The three models are based on a SSB mechanism due to the top Yukawa interaction. However the starting assumption of the two models of ref.[4] is different: in these two models instead of having no mass term ($\mu^2 = 0$) and a quartic term ($\lambda \neq 0$), there is no quartic term ($\lambda = 0$) but a mass term ($\mu^2 \neq 0$).

Let us now consider the model. In the standard model considered as an effective theory with the Higgs potential of Eq.(1) the effective potential is

$$V_{eff}(\phi) = \frac{\lambda}{4}\phi^4 + \frac{1}{32\pi^2} \int_0^{\Lambda^2} dq^2 q^2 \cdot \{A(3\lambda) + 3A(\lambda) + 6A(\frac{g_2^2}{4}) + 3A(\frac{g_1^2 + g_2^2}{4}) - 4N_c A(\frac{g_t^2}{2})\}$$

$$\begin{aligned}
&= \frac{\lambda}{4}\phi^4 + \frac{1}{32\pi^2} \cdot \{I(3\lambda) + 3I(\lambda) + 6I(\frac{g_2^2}{4}) + 3I(\frac{g_1^2 + g_2^2}{4}) - 4N_c(\frac{g_t^2}{2})\} \\
\end{aligned} \tag{3}$$

with:

$$A(z) = \ln(1 + \frac{z\phi^2}{q^2}) \tag{4}$$

$$I(z) = \frac{1}{2}[\Lambda^4 \ln(1 + \frac{z\phi^2}{\Lambda^2}) - z^2\phi^4 \ln(1 + \frac{\Lambda^2}{z\phi^2}) + \Lambda^2 z\phi^2]. \tag{5}$$

In Eq.(3) we have neglected all the Yukawa forces except for the top quark. g_2, g_1, g_t are the gauge couplings and the top Yukawa coupling respectively. In our normalization the W, Z and top masses are given by $m_W^2 = g_2^2 v^2/4, m_Z^2 = (g_1^2 + g_2^2)v^2/4, m_t^2 = g_t^2 v^2/2$ with $v \simeq 246$ GeV. In the numerical results the values $m_Z = 91.19$ GeV and $m_W = 80.28$ GeV will be used.

There is a range of values of Λ and λ such that the effective potential has an extremum in $\phi = v$. This range is defined by the equation:

$$\begin{aligned}
0 &= \frac{\partial V_{eff}(\phi)}{\partial \phi} \Big|_{\phi=v} = \lambda(v^3 + \frac{3v}{8\pi^2}\Lambda^2) \\
&+ \frac{1}{8\pi^2 v} \{ \frac{1}{2}[-9\lambda^2 v^4 \ln(1 + \frac{\Lambda^2}{3\lambda v^2})] + \frac{3}{2}[-\lambda^2 v^4 \ln(1 + \frac{\Lambda^2}{\lambda v^2})] \} \\
&+ \frac{1}{8\pi^2 v} \{ -2N_c[m_t^2 \Lambda^2 - m_t^4 \ln(1 + \frac{\Lambda^2}{m_t^2})] \\
&\quad + 3[m_W^2 \Lambda^2 - m_W^4 \ln(1 + \frac{\Lambda^2}{m_W^2})] \\
&\quad + \frac{3}{2}[m_Z^2 \Lambda^2 - m_Z^4 \ln(1 + \frac{\Lambda^2}{m_Z^2})] \}
\end{aligned} \tag{6}$$

The numerical solution of Eq.(6) for λ is represented as a function of Λ in Fig.1. From Eq.(6) it is easy to see that for $\Lambda \rightarrow \infty, \lambda$ goes to the asymptotical value

$$\lambda \xrightarrow{\Lambda \rightarrow \infty} (g_t^2 - \frac{1}{4}g_2^2 - \frac{1}{8}(g_1^2 + g_2^2)) \tag{7}$$

For $m_t = 180$ GeV it corresponds to $\lambda = 0,90$.

From the numerical solution of Eq.(6) and from the definition

$$\begin{aligned}
m_H^2 &= \left(\frac{\partial^2 V_{eff}(\phi)}{\partial \phi^2} \right) \Big|_{\phi=v} = \lambda(3v^2 + \frac{3}{8\pi^2}\Lambda^2) \\
&+ \frac{1}{8\pi^2 v^2} \{ \frac{1}{2}[\frac{18\lambda^2 v^4}{\Lambda^2 + 3\lambda v^2}\Lambda^2 - 27\lambda^2 v^4 \ln(1 + \frac{\Lambda^2}{3\lambda v^2})]
\end{aligned}$$

$$\begin{aligned}
& + \frac{3}{2} \left[\frac{2\lambda^2 v^4}{\Lambda^2 + \lambda v^2} \Lambda^2 - 3\lambda^2 v^4 \ln\left(1 + \frac{\Lambda^2}{\lambda v^2}\right) \right] \} \\
& + \frac{1}{8\pi^2 v^2} \left\{ -2N_c \left[\left(m_t^2 + \frac{2m_t^4}{\Lambda^2 + m_t^2} \right) \Lambda^2 - 3m_t^4 \ln\left(1 + \frac{\Lambda^2}{m_t^2}\right) \right] \right. \\
& \quad + 3 \left[\left(m_W^2 + \frac{2m_W^4}{\Lambda^2 + m_W^2} \right) \Lambda^2 - 3m_W^4 \ln\left(1 + \frac{\Lambda^2}{m_W^2}\right) \right] \\
& \quad \left. + \frac{3}{2} \left[\left(m_Z^2 + \frac{2m_Z^4}{\Lambda^2 + m_Z^2} \right) \Lambda^2 - 3m_Z^4 \ln\left(1 + \frac{\Lambda^2}{m_Z^2}\right) \right] \right\} \quad (8)
\end{aligned}$$

we can now obtain the corresponding Higgs mass which is represented in Fig.2 as a function of Λ for $m_t = 180$ GeV. From Fig.2 we see there is SSB for $250 \text{ GeV} \lesssim \Lambda \lesssim 10^{7.8} \text{ GeV}$ and the predicted Higgs mass is bigger than the experimental lower limit, $m_H \gtrsim 65 \text{ GeV}$, for $350 \text{ GeV} \lesssim \Lambda \lesssim 10^{7.6} \text{ GeV}$. It can be shown for these ranges of values of Λ that the effective potential is a double well potential bounded from below. In addition, and this is the important predictive result of the model, whatever the value of Λ is, the Higgs mass is lighter than a relatively low upper limit: $m_H \lesssim 300 \text{ GeV}$.

An important characteristic of the model presented here is that in Eq.(8) there is no quadratic term in Λ for $\Lambda \rightarrow \infty$. That can be understood easily from Eq.(7) which is the condition of cancellation of quadratic divergences in the ordinary standard model [5]. The quadratic divergences come from the tadpole diagrams of the scalar field. The extremum condition (Eq.(6)) is, by definition of the effective potential, the condition which imposes to the scalar field to have a zero one-point function and consequently, in a effective theory, to have no quadratic divergences in Eq.(8). This cancellation of quadratic terms in Λ explains the relatively weak dependance of m_H^2 on Λ .

An other important characteristic of the model is that, for values of Λ sizeably bigger than 1 TeV, the higher orders scalar contributions (of the order $\lambda^3, \lambda^4, \dots$) are expected to be big. Indeed if we don't take into account the λ^2 order terms in Eqs.(6), (8), we obtain for m_H a result which for $\Lambda \gg 1 \text{ TeV}$ is sizeably different from the result we obtain when we don't take into account the λ^2 order terms (see Fig.2). So, for values of $\Lambda \gg 1 \text{ TeV}$, the validity of the perturbation theory is doubtful and the one loop approximation is expected to break down. In the following we will discuss the results only for $\Lambda \lesssim 10 \text{ TeV}$ ¹.

¹If we consider as serious the problem of fine-tuning, related to the cancellation of quadratic

In Fig.3 we plot with better accuracy m_H^2 coming from Eqs.(6), (8), for $\Lambda \leq 10$ TeV. To give an idea, if $\Lambda = 0.5$ TeV, 1 TeV, 5 TeV, 10 TeV, we get from Eqs.(6), (8), $m_H = 112$ GeV, 206 GeV, 297 GeV, 289 GeV. If we don't take into account the λ^2 order terms we get from Eqs.(6), (8):

$$\begin{aligned}
m_H^2 = & \frac{1}{8\pi^2 v^2} \left\{ -2N_c m_t^4 \left[\frac{2\Lambda^2}{\Lambda^2 + m_t^2} - 2 \ln\left(1 + \frac{\Lambda^2}{m_t^2}\right) \right] \right. \\
& + 3m_W^4 \left[\frac{2\Lambda^2}{\Lambda^2 + m_W^2} - 2 \ln\left(1 + \frac{\Lambda^2}{m_W^2}\right) \right] \\
& \left. + \frac{3}{2} m_Z^4 \left[\frac{2\Lambda^2}{\Lambda^2 + m_Z^2} - 2 \ln\left(1 + \frac{\Lambda^2}{m_Z^2}\right) \right] \right\} \\
& - \frac{1}{4\pi^2} \frac{1}{(v^2 + \frac{3}{8\pi^2} \Lambda^2)} \left\{ 2N_c m_t^4 \ln\left(1 + \frac{\Lambda^2}{m_t^2}\right) \right. \\
& - 3m_W^4 \ln\left(1 + \frac{\Lambda^2}{m_W^2}\right) \\
& \left. - \frac{3}{2} m_Z^4 \ln\left(1 + \frac{\Lambda^2}{m_Z^2}\right) \right\} \\
& + \frac{1}{4\pi^2} \frac{\Lambda^2}{(v^2 + \frac{3}{8\pi^2} \Lambda^2)} (2N_c m_t^2 - 3m_W^2 - \frac{3}{2} m_Z^2) \quad (9)
\end{aligned}$$

For $\Lambda = 0.5$ TeV, 1 TeV, 5 TeV, 10 TeV, Eq.(9) gives $m_H = 115$ GeV, 206 GeV, 339 GeV, 352 GeV respectively. In Eq.(9), the third term is dominant: it gives $m_H = 121$ GeV, 204 GeV, 319 GeV, 326 GeV respectively.² This third term of Eq.(9) is consequently a good approximation (valid at $\simeq 10\%$) of the complete one loop result for $\Lambda \lesssim 10$ TeV. For $\Lambda^2 < \frac{8\pi^2}{3} v^2 (\simeq (1260 \text{ GeV})^2)$ and for $\Lambda^2 > \frac{8\pi^2}{3} v^2$ the third term of Eq.(9) can be written as

$$m_H^2 \stackrel{\Lambda^2 < 8\pi^2 v^2/3}{\simeq} \frac{1}{4\pi^2} \frac{\Lambda^2}{v^2} (2N_c m_t^2 - 3m_W^2 - \frac{3}{2} m_Z^2) + \mathcal{O}(\frac{3\Lambda^2}{8\pi^2 v^2}) \quad (10)$$

$$m_H^2 \stackrel{\Lambda^2 > 8\pi^2 v^2/3}{\simeq} (\frac{4}{3} N_c m_t^2 - 2m_W^2 - m_Z^2) + \mathcal{O}(\frac{8\pi^2 v^2}{3\Lambda^2}) \quad (11)$$

$$\stackrel{m_t=180 \text{ GeV}}{\simeq} (329 \text{ GeV})^2 + \mathcal{O}(\frac{8\pi^2 v^2}{3\Lambda^2}) \quad (12)$$

Eq.(10) explains why m_H increases quickly in Fig.3 for small value of Λ . Eq.(11) explains the plateau in Fig.3 for $\Lambda \gtrsim 2$ TeV. Indeed, for $\Lambda^2 > (8\pi^2 v^2/3)$, Eq.(11) shows that up to small corrections (the correction of the order $8\pi^2 v^2/3\Lambda^2$, the two

divergences, let us recall that to take Λ of the order of 1 TeV avoids this problem.

²Note here that for $\Lambda \simeq 1$ TeV the third term of Eq.(9) is an excellent approximation of the full one-loop result.

first terms in Eq.(9) and the λ^2 order terms which together give for example, for $\Lambda = 3 \text{ TeV}$, a $\simeq 10\%$ correction), m_H^2 can be reduced to the expression of Eq.(11). Neglecting the $\mathcal{O}(8\pi^2 v^2/3\Lambda^2)$ term, Eq.(11) is the equation obtained by imposing the cancellation of quadratic divergences in the ordinary standard model at lowest order [5]. This result can be understood in the calculation by the fact that the coefficients of the $\Lambda^2/4\pi^2(v^2 + 3\Lambda^2/8\pi^2)$ term in the dominant third term of Eq.(9) come from the coefficients of the quadratically divergent terms in Eqs.(3)-(5).

Note also that in the third term of Eqs.(9)-(11) we see clearly that a heavy top is an essential ingredient for the quantum SB mechanism proposed here. A real Higgs boson requires:

$$m_t^2 \gtrsim (\frac{1}{2}m_W^2 + \frac{1}{4}m_Z^2) \simeq (73\text{GeV})^2 \quad (13)$$

In Eq.(13), the masses are squared because the contribution of each sector in the dominant third term of Eq.(9) is proportional to the corresponding squared mass. This explains that the gauge bosons contribution is a small effect with respect to the top Yukawa contribution but is not negligible.³ Eq.(9) without the gauge bosons contributions gives for $\Lambda = 0,5 \text{ TeV}$, 1 TeV , 5 TeV , 10 TeV , $m_H = 126 \text{ GeV}$, 226 GeV , 368 GeV , 382 GeV respectively, to be compared with $m_H = 115 \text{ GeV}$, 206 GeV , 339 GeV , 352 GeV when we take the full Eq.(9).

To conclude, we propose, starting from an effective theory without a scalar mass term, a dynamical SSB mechanism which is due to the large top Yukawa force. We obtain the relatively low upper limit: $m_H \lesssim 300 \text{ GeV}$. We have $m_H \gtrsim 100 \text{ GeV}$ if $\Lambda \gtrsim 0.5 \text{ TeV}$ and over a wide range of values of Λ ($2 \text{ TeV} \lesssim \Lambda \lesssim 10 \text{ TeV}$) we obtain $m_H \simeq 290 \text{ GeV}$.

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³In the models of ref.[4] the contribution of each sector is proportional to the 4th power of the mass. In this case the gauge bosons contribution is negligible.

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Figure captions

Fig.1 λ as a function of Λ from Eq.(6) for $m_t=180$ GeV.

Fig.2 m_H as a function of Λ from Eqs.(6), (8) for $m_t = 180$ GeV taking into account the λ^2 order terms (solid line) and without taking into account the λ^2 order terms (dashed line).

Fig.3 m_H as a function of Λ from Eqs.(6), (8) taking into account the λ^2 order terms for $\Lambda \leq 10$ TeV and for $m_t=168$ GeV, 180 GeV, 192 GeV.

$$m_t = 192 \text{ GeV}$$

$$m_t = 192 \text{ GeV}$$

$$m_t = 180 \text{ GeV}$$

$$m_t = 180 \text{ GeV}$$

$$m_t = 168 \text{ GeV}$$

$$m_t = 168 \text{ GeV}$$

$$m_H \text{ (GeV)}$$

$$m_H \text{ (GeV)}$$

$$m_H \text{ (GeV)}$$

$$m_H \text{ (GeV)}$$

$$\lambda$$

$$\lambda$$

$$\log(\Lambda/1\text{GeV})$$

$$\log(\Lambda/1\text{GeV})$$

$$\log(\Lambda/1\text{GeV})$$

$$\log(\Lambda/1\text{GeV})$$

$$\Lambda \text{ (TeV)}$$

$$\Lambda \text{ (TeV)}$$

$$\Lambda \text{ (TeV)}$$

$$\Lambda \text{ (TeV)}$$